Find the roots and solutions of $f(x) = x^2 + 4x - 6$

Can’t straight up factor this, it is a quadratic so use the quadratic formula…

$$x^2 + 4x - 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1, b = 4, c = -6$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-6)}}{2(1)} = \frac{-4 \pm \sqrt{16 + 24}}{2} = \frac{-4 \pm \sqrt{40}}{2} = \frac{-4 \pm 2\sqrt{10}}{2} = -2 \pm \sqrt{10}$$

Note there are two solutions (the ±)
…they come in pairs
…there is the plus version and the minus version

We call these “***conjugates***”

…these are irrational conjugates

Find the roots and solutions of $f(x) = x^2 + 4x + 5$

Can't straight up factor this, it is a quadratic so use the quadratic formula…

$$x^2 + 4x + 5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1, b = 4, c = 5$$

Note there are two solutions (the ±)
…they come in pairs
…there is the plus version and the minus version

…these are complex conjugates

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$$

LESSON 4.5c and 4.6

Irrational/Complex Conjugate Theorem

Fundamental Theorem of Algebra

Today you will:

- Create/write polynomials given their roots
- Practice using English to describe math processes and equations

Core Vocabulary:

- Conjugate
- Irrational Conjugates Theorem, p. 193
- Complex Conjugates Theorem, p. 199
- Fundamental Theorem of Algebra, p. 198

Prior:

- Real numbers
- Irrational numbers
- Complex/imaginary numbers

Core Concept

The Irrational Conjugates Theorem

Let f be a polynomial function with rational coefficients, and let a and b be rational numbers such that $\sqrt{b}$ is irrational. If $a + \sqrt{b}$ is a zero of f, then $a - \sqrt{b}$ is also a zero of f.

Core Concept

The Complex Conjugates Theorem

If f is a polynomial function with real coefficients, and $a + bi$ is an imaginary zero of f, then $a - bi$ is also a zero of f.

Bottom line:

Irrational and complex roots ***always*** come in conjugate pairs.

Core Concept

The Fundamental Theorem of Algebra

Theorem If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has at least one solution in the set of complex numbers.

Corollary If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has exactly n solutions provided each solution repeated twice is counted as two solutions, each solution repeated three times is counted as three solutions, and so on.

STUDY TIP

The statements "the polynomial equation $f(x) = 0$ has exactly n solutions" and "the polynomial function f has exactly n zeros" are equivalent.

Bottom Line:
A degree n polynomial has exactly n zeros/roots/factors (counting irrational and complex solutions)

Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the zeros 3 and $2 + \sqrt{5}$.

SOLUTION

Because the coefficients are rational and $2 + \sqrt{5}$ is a zero, $2 - \sqrt{5}$ must also be a zero by the Irrational Conjugates Theorem. Use the three zeros and the Factor Theorem to write $f(x)$ as a product of three factors.

$f(x) = (x - 3)[x - (2 + \sqrt{5})][x - (2 - \sqrt{5})]$	Write $f(x)$ in factored form.
$= (x - 3)[(x - 2) - \sqrt{5}][(x - 2) + \sqrt{5}]$	Regroup terms.
$= (x - 3)[(x - 2)^2 - 5]$	Multiply.
$= (x - 3)[(x^2 - 4x + 4) - 5]$	Expand binomial.
$= (x - 3)(x^2 - 4x - 1)$	Simplify.
$= x^3 - 4x^2 - x - 3x^2 + 12x + 3$	Multiply.
$= x^3 - 7x^2 + 11x + 3$	Combine like terms.

$= x^3 - 7x^2 + 11x + 3$ Combine like terms.

Check

You can check this result by evaluating f at each of its three zeros.

$$f(3) = 3^3 - 7(3)^2 + 11(3) + 3 = 27 - 63 + 33 + 3 = 0 \quad \checkmark$$

$$f(2 + \sqrt{5}) = (2 + \sqrt{5})^3 - 7(2 + \sqrt{5})^2 + 11(2 + \sqrt{5}) + 3$$

$$= 38 + 17\sqrt{5} - 63 - 28\sqrt{5} + 22 + 11\sqrt{5} + 3$$

$$= 0 \quad \checkmark$$

Because $f(2 + \sqrt{5}) = 0$, by the Irrational Conjugates Theorem $f(2 - \sqrt{5}) = 0$. ✓

Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the zeros 2 and $3 + i$.

SOLUTION

Because the coefficients are rational and $3 + i$ is a zero, $3 - i$ must also be a zero by the Complex Conjugates Theorem. Use the three zeros and the Factor Theorem to write $f(x)$ as a product of three factors.

$f(x) = (x - 2)[x - (3 + i)][x - (3 - i)]$	Write $f(x)$ in factored form.
$= (x - 2)[(x - 3) - i]\,[(x - 3) + i]$	Regroup terms.
$= (x - 2)[(x - 3)^2 - i^2]$	Multiply.
$= (x - 2)[(x^2 - 6x + 9) - (-1)]$	Expand binomial and use $i^2 = -1$.
$= (x - 2)(x^2 - 6x + 10)$	Simplify.
$= x^3 - 6x^2 + 10x - 2x^2 + 12x - 20$	Multiply.
$= x^3 - 8x^2 + 22x - 20$	Combine like terms.

$= x^3 - 8x^2 + 22x - 20$ Combine like terms.

Check

You can check this result by evaluating *f* at each of its three zeros.

$f(2) = (2)^3 - 8(2)^2 + 22(2) - 20 = 8 - 32 + 44 - 20 = 0$ ✓

$f(3 + i) = (3 + i)^3 - 8(3 + i)^2 + 22(3 + i) - 20$

$= 18 + 26i - 64 - 48i + 66 + 22i - 20$

$= 0$ ✓

Because $f(3 + i) = 0$, by the Complex Conjugates Theorem $f(3 - i) = 0$. ✓

Homework

Pg 195, #41-51

Pg 202, #21-28